WEEKLY TEST TYJ -1 TEST - 22 R
SOLUTION Date 22-09-2019

[PHYSICS]

1. Initial kinetic energy of mass

$$K_{\text{initial}} = \frac{1}{2} I\omega^2 = \frac{1}{2}(mr^2)\omega^2 = \frac{1}{2}m(r^2\omega^2)$$

$$= \frac{1}{2}mv_0^2$$

Conservation of angular momentum

$$mv_0R_0 = mv\frac{R_0}{2} \quad \Rightarrow \quad v = 2v_0$$

Final kinetic energy of mass

$$K_{\text{final}} = \frac{1}{2}mv^2 = \frac{1}{2}m(2v_0)^2 = 4.\frac{1}{2}mv_0^2 = 4k_{\text{initial}}$$

2. $\vec{L} = \vec{r} \times \vec{p}$

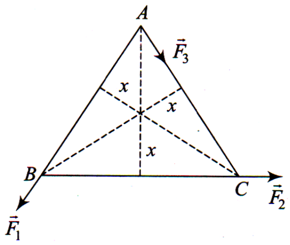
By right-hand screw, rule, the direction of $\vec{L}$ is perpendicular to the plane containing $\vec{r}$ & $\vec{p}$.

The mass is rotating in the plane, about a fixed point, thus this plane will contain $\vec{r}$ & $\vec{p}$ and the direction of $\vec{L}$ will be perpendicular to this plane.

3. If we take boat and both persons as a system, there is no external force acting on the system. The center of mass of the system is initially at rest and will be at rest as there is no external force acting on it to displace center of mass. Hence there is no shifting of center of mass.

4. Taking torque about O, net torque should be zero.

$$F_2 \times x - F_3 \times x + F_1 \times x = 0$$



$$F_3 = F_1 + F_2$$

5. From conservation of angular momentum

$$I\omega = mvr$$

$$200 \times \omega = 50 \times 2 \times 1$$

$$\omega = \frac{1}{2} \text{ rad/s}$$

$$v = r\omega = 1 \text{ m/s}$$

$$\therefore \quad T = \frac{2\pi r}{1-(-1)} = \frac{2\pi r}{2} = \pi r = 2\pi \text{ s}$$

6. Moment of inertia of circular disc = $\frac{1}{2}mR^2$. Thus, as the distance between the centre and the point increases, the moment of inertia increases.

7. $$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{300(0) + 500(40) + 400(70)}{300 + 500 + 400}$$

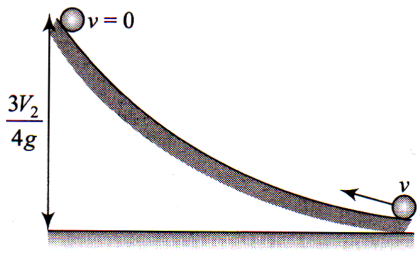
$$= \frac{20000 + 28000}{1200} = \frac{48000}{1200} = 40 \text{ cm}$$

8. $$\frac{1}{2} \times 3(4)^2 + \frac{1}{2} \times \frac{(3 \times R^2)}{2} \times \left(\frac{4}{R}\right)^2 = \frac{1}{2}Kx^2$$

$\Rightarrow \quad x = 0.6$ m

9. From law of conservation of mechanical energy

$$\Delta K + \Delta U = 0$$



$$\left[0 - \left(\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2\right)\right] + \left(mg \times \frac{3v^2}{4g}\right) = 0$$

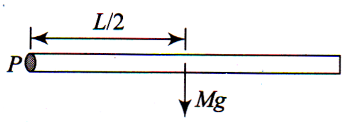
$$\Rightarrow \quad \frac{1}{2}I\omega^2 = \frac{3}{4}mv^2 - \frac{1}{2}mv^2 = \frac{mv^2}{2}\left(\frac{3}{2} - 1\right)$$

As cylinder is rolling $\omega = \frac{v}{R}$

or $\quad \frac{1}{2}I\frac{v^2}{R^2} = \frac{mv^2}{4} \quad$ or $\quad I = \frac{1}{2}mR^2$

Hence, object is a disc.

10. Taking torque about P



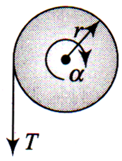
$$Mg\,\frac{L}{2}=\left(\frac{ML^2}{3}\right)\alpha$$

Hence $\alpha=\dfrac{3g}{2L}$

11. Applying torque equation about center of cylinder $\tau = I\alpha$

α is the angular acceleration of cylinder and it is given

$$\alpha = 2\ \text{revolution/s}^2 = 2\times 2\pi = 4\pi\ \text{rad/s}^2$$



$$Tr = I\alpha$$

$$T = \frac{I\alpha}{r}=\frac{mr^2}{2}\times\frac{\alpha}{r}=\frac{mr\alpha}{2}$$

$$=\frac{50\times 0.5\times 4\pi}{2}\ \text{N} = 157\ \text{N}$$

12. Acceleration of sphere when it is slipping down the incline , $a_{\text{slipping}} = g\sin\theta$

Acceleration of sphere when it is rolling down

$$a_{\text{rolling}}=\frac{g\sin\theta}{1+\dfrac{K^2}{r^2}}=\frac{5}{7}g\sin\theta$$

Hence required ratio $\dfrac{a_{\text{rolling}}}{a_{\text{slipping}}}=\dfrac{5}{7}$

13. Initial angular momentum $L_{\text{initial}} = mv_0R$

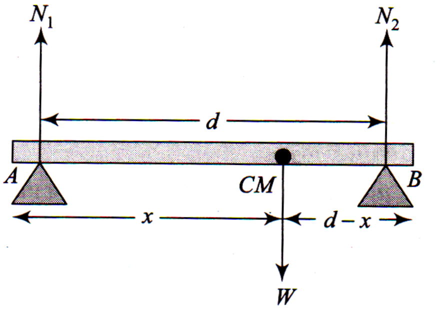
Initial angular momentum $L_{\text{final}} = mv\dfrac{R}{2}$

Conservation of angular momentum

$$mv_0R_0 = mv\frac{R_0}{2} \quad\Rightarrow\quad v = 2v_0$$

$$\text{KE}=\frac{1}{2}mv^2 = 2\,mv_0^2$$

14. Taking torque about end A

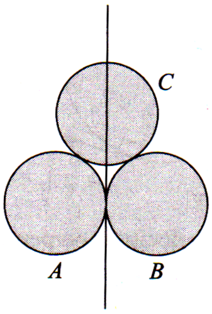


$$\tau_B = 0$$

$$\Rightarrow \quad N_1 d = W(d-x)$$

$$\Rightarrow \quad N_1 = \frac{W(d-x)}{d}$$

15. $I = I_A + I_B + I_C$



$$= \left(\frac{2}{3}mr^2 + mr^2\right) + \left(\frac{2}{3}mr^2 + mr^2\right) + \frac{2}{3}mr^2$$

$$= 4\,mr^2$$